

Probability and Measure Theory Review



Probability
Basic

Inequalities

4-Dummies

Measure Theory

Limits

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Probability Basic for 100.



The space of events \mathcal{F} for Ω satisfies:

$$\Omega \notin \mathcal{F}$$

$$\emptyset \in \mathcal{F}$$

$$A \in \mathcal{F} \Rightarrow A \in \mathcal{F}^C$$

$$A_1, A_2, \dots, A_n, \dots \in \mathcal{F} \Rightarrow \bigcup_{n=1}^{\infty} A_n \in \mathcal{F}$$

none of them

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Probability Basic for 200.



A probability P on (Ω, \mathcal{F}) satisfies

$$P(\Omega) \geq 1.2$$

$$P(A \cup B) = P(A) \cup P(B) - P(A \cap B)$$

For disjoint sets in \mathcal{F} ,
$$P\left(\bigcup_{n=1}^{\infty} A_n\right) = \sum_{n=1}^N P(A_n)$$

$$P(\emptyset) < 0$$

none of them

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Probability Basic for 300.



Conditional probability $P(A|B)$ equals

$$P(A \cap B)$$

$$P(A) \cdot P(B)$$

$$(P(B) \times 1/P(A \cap B))^{-1}$$

$$P(A) + P(B) - P(A \cap B)$$

$$P(A - B)$$

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Probability Basic for 400.



X is a random variable iff

$$\forall \alpha \in \mathbb{R} \quad \{\omega : X(\omega) \geq \alpha\} \notin \mathbb{X}$$

$$\exists \alpha \in \mathbb{R} \quad \{\omega : X(\omega) \geq \alpha\} \in \mathbb{X}$$

$$\exists \alpha \in \mathbb{R} \quad \{\omega : X(\omega) < \alpha\} \in \mathbb{X}$$

$$\forall \alpha \in \mathbb{R} \quad \{\omega : X(\omega) = \alpha\} \in \mathbb{X}$$

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Inequalities for 100.

Boole's Inequality states that

$E[g(X)] \geq g(E(X))$ for g convex

$$P\left(\bigcup_{n=1}^{\infty} A_n\right) \leq \sum_{n=1}^{\infty} P(A_n)$$

$$P(|X - \mu| \geq a) \leq \frac{\sigma^2}{a^2}$$

$$P(|X| \geq a) \leq \frac{1}{a} E(|X|)$$

none of them

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Inequalities for 200.



Markov Inequality states that

$$P(\Omega) \geq 1.2$$

$$P(A \cup B) = P(A) \cup P(B) - P(A \cap B)$$

For disjoint sets in \mathcal{F} ,
$$P\left(\bigcup_{n=1}^{\infty} A_n\right) = \sum_{n=1}^N P(A_n)$$

$$P(\emptyset) < 0$$

$$P(|X| \geq a) \leq \frac{1}{a} E(|X|)$$



Inequalities for 300.

Chebyshev's inequality concludes that

$E[g(X)] \geq g(E(X))$ for g convex

$$P\left(\bigcup_{n=1}^{\infty} A_n\right) \leq \sum_{n=1}^N P(A_n)$$

$$P(|X - \mu| \geq a) \leq \frac{\sigma^2}{a^2}$$

$$P(|X| \geq a) \leq \frac{1}{a} E(|X|)$$

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Inequalities for 400.



Jensen Inequality states that

$E[g(X)] \geq g(E(X))$ for g convex

$$P\left(\bigcup_{n=1}^{\infty} A_n\right) \leq \sum_{n=1}^N P(A_n)$$

$$P(|X - \mu| \geq a) \leq \frac{\sigma^2}{a^2}$$

$$P(|X| \geq a) \leq \frac{1}{a} E(|X|)$$

none of them

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4-Dummies for 100.



What is $(A^c)^c$?

\emptyset

X

$\{A\}$

$A - X$

$X - A$

X^c

none of them

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4-Dummies for 200.



The complement of $\bigcap_{m=1}^{\infty} E_m$ is

$$\bigcup_{m=1}^{\infty} E_m$$

$$\bigcap_{m=1}^{\infty} E_m^c$$

$$\bigcap_{m=1}^{\infty} E_m^c$$

$$\bigcup_{n=1}^{\infty} E_n^c$$

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4-Dummies for 300.



If X is a set, what is 2^X ?

2 raised to the power X

2 times 2 times 2 ... X times

The X power of 2

all subsets of X

I do not give a dime

The power rangers

none of them

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4-Dummies for 400.



In Measure Theory, what does LDCT mean ?

Lower Derivative Control Theorem

Least Derivative Common Term

Lightweight Digital Command Terminal

Lebesgue's Dominated Convergence Theorem

Leibniz Derivative Convergence Theory

Lagrange D' Cumulative Term

Laplace Derivative Continuous Theorem

none of them

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Measure Theory for 100.



If (X, \mathcal{F}, μ) is a measure space and $x \in A$, what is the value of $\chi_A(x)$?

-1

0

1

x

Ax

$\mu(A)$

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Measure Theory for 200.



If (X, \mathcal{F}, μ) is a measure space and $A \in \mathcal{F}$, what is the value of $\int \chi_A d\mu$?

-1

0

1

x

Ax

$\mu(A)$

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Measure Theory for 300.



What is the Lebesgue measure?

It is about the Riemann integral

It is the integral

To each interval assigns its length

$$P(A \cap B) = P(A) \cdot P(B)$$

$$P(|X| \geq a) \leq \frac{1}{a} E(|X|)$$

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Measure Theory for 400.



What does Radon Nikodym theorem conclude?

$$\frac{X_1 + X_2 + \cdots + X_n}{n} \longrightarrow \mu$$

$$\lim \int f_n d\mu = \int f d\mu$$

there is f with $\lambda(A) = \int_A f d\mu$

there is a measure that extends to a sigma algebra

the moment generating function

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Limits for 100.

CLT states that if X_n are i.i.d.r.v. with mean 0 and variance 1, then , $\frac{X_1 + X_2 + \dots + X_n}{\sqrt{n}}$ converges to

μ

1

normal random variable mean 0, variance 1

Of course the mean

the variance?

X_1

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Limits for 200.

LDCT concludes that $\lim_{n \rightarrow \infty} \int f_n d\mu = \int f d\mu$. What are the hypotheses?

$$0 \leq f_n \leq f_{n+1}, f_n \text{ measurable}$$

$$0 \leq f_n \leq f_{n+1}, f_n \rightarrow f, f_n \text{ measurable}$$

$$0 \leq f_n \rightarrow f, f_n \text{ measurable}$$

$$g \geq |f_n| \rightarrow f, f_n \text{ measurable, } g \text{ integrable}$$

f_n increasing sequence of measurable functions
converging to f



Limits for 300.

If $0 \leq f_n \leq f_{n+1}$ are measurable and converge to f ,
MCT states

$$\frac{X_1 + X_2 + \cdots + X_n}{\sqrt{n}} \rightarrow 1$$

$$P\left(\bigcup_{n=1}^{\infty} A_n\right) \leq \sum_{n=1}^{\infty} P(A_n)$$

$$\frac{X_1 + X_2 + \cdots + X_n}{n} \rightarrow \mu$$

$$\lim_{n \rightarrow \infty} \int f_n d\mu = \int f d\mu.$$

$$P(|X| \geq a) \leq \frac{1}{a} E(|X|)$$



Limits for 400.

If $0 \leq f_n$ are measurable, Fatou's lemma states

$$\int \liminf f_n d\mu \leq \liminf \int f_n d\mu$$

$$\frac{X_1 + X_2 + \cdots + X_n}{n} \longrightarrow \mu$$

$$\lim_{n \rightarrow \infty} \int f_n d\mu = \int f d\mu.$$

$$\frac{X_1 + X_2 + \cdots + X_n}{\sqrt{n}} \longrightarrow 1$$

there is f with $\lambda(A) = \int_A f d\mu$

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