# Sequences, Metrics and Topology

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|---|---|---|---|---|---|
| 100 | 100 | 100 | 100 | 100 | 100 |
| 200 | 200 | 200 | 200 | 200 | 200 |
| 300 | 300 | 300 | 300 | 300 | 300 |
| 400 | 400 | 400 | 400 | 400 | 400 |

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Definitions for 100.

In a topological space $\{p_n\}$ **converges to a point** $p$ iff

- a $\exists O \in \tau \quad O \ni p \quad \forall N \in \mathbb{N} \quad \forall n \geq N \quad p_n \in O$
- b $\exists O \in \tau \quad O \ni p_n \quad \exists N \in \mathbb{I} \quad \forall n \geq N \quad p \in O$
- c $\forall O \in \tau \quad O \ni p_n \quad \exists N \in \mathbb{N} \quad \forall n \geq N \quad p \in O$
- d $\forall O \in \tau \quad O \ni p \quad \exists N \in \mathbb{N} \quad \forall n \geq N \quad p_n \in O$
- e $\forall r > 0 \quad \exists N \in \mathbb{N} \quad \forall n \geq N \quad d(p_n, p) < r$
- f none of them

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Definitions for 200.

In a metric space, the sequence $\{p_n\}$ **converges to a point** $p$ iff

a $\forall O \in \tau \quad O \ni p \quad \exists N \in \mathbb{N} \quad \forall n \geq N \quad p_n \in O$

b $\exists r > 0 \; \exists N \in \mathbb{N} \quad \forall n < N \quad d(p_n, p) < r$

c $\exists r > 0 \quad \exists N \in \mathbb{N} \quad \forall n \geq N \quad d(p_n, p) > r$

d $\forall r > 0 \quad \exists N \in \mathbb{N} \quad \forall n < N \quad d(p_n, p) > r$

e $\exists r > 0 \quad \forall N \in \mathbb{N} \quad \exists n < N \quad d(p_n, p) > r$

f $\forall r > 0 \quad \exists N \in \mathbb{N} \quad \forall n \geq N \quad d(p_n, p) < r$

Definitions for 300.

We say that $\{p_{n_i}\}_i$ is a **subsequence** of $\{p_n\}_n$ if

a $n_1 > n_2 > n_3 > \cdots$

b $n_1 \leq n_2 \leq n_3 \leq \cdots$

c $p_{n_1} < p_{n_2} < p_{n_3} < \cdots$

d $p_1 < p_1 < p_3 < \cdots$

e $p_{n_1} > p_{n_2} > p_{n_3} > \cdots$

f $p_1 > p_1 > p_3 > \cdots$

g $n_1 < n_2 < n_3 < \cdots$

h none of them

## Definitions for 400.

We say that $\{p_n\}$ is a **Cauchy sequence** if

- a $\forall r > 0, \exists N \in \mathbb{N}, \exists n, m \geq N, d(p_n, p_m) < r$
- b $\forall r > 0, \exists N \in \mathbb{N}, \forall n, m \geq N, d(p_n, p_m) < r$
- c $\exists r > 0, \exists N \in \mathbb{N}, \exists n, m \geq N, d(p_n, p_m) < r$
- d $\exists r > 0, \exists N \in \mathbb{N}, \forall n, m \geq N, d(p_n, p_m) < r$
- e $\forall r > 0, \forall N \in \mathbb{N}, \exists n, m \geq N, d(p_n, p_m) < r$
- f none of them

Examples for 100.

In $R$ usual top, $p_n = \frac{1}{n}$ converges to

a $3^0 - \frac{1+2+3}{3^2 - 3^1}$

b 1

c Doesn't converge, 0 is not in our universe

d 2

e $\infty$

f none of them

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## Examples for 200.

$X = (0, \infty)$, usual topology $p_n = \dfrac{1}{n}$ converges to

- a 0
- b .5
- c 1
- d 2
- e $\infty$
- f none of them

Examples for 300.

In $R$ with $\tau = \{\text{all sets}\}$, the sequence $\frac{1}{n}$ converges to

- a 0
- b .5
- c 1
- d 2
- e $\infty$
- f none of them

Examples for 400.

In $R$ with $\tau = \{\emptyset, \mathbb{R}\}$ the sequence $\frac{1}{n}$ converges to

| | |
|---|---|
| a | 0 |
| b | .5 |
| c | 1 |
| d | 2 |
| e | $\infty$ |
| f | All real numbers |
| g | none of them |



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## Theorems for 100.

If $p_n$ is Cauchy, find the incorrect answer

- a $p_n$ is for sure bounded.
- b $p_n$ it might not converge.
- c $p_n$ converges provided it has a convergent subseq.
- d $p_n$ converges if it lives in a compact set.
- e $p_n$ converges if it lives in $\mathbb{R}^k$.
- f $p_n$ might jump b&f between two values

# Theorems for 200.

If $p_n \to p$, then

- a  Any subsequence of $p_n$ converges to $p$
- b  There is one subsequence of $p_n$ that diverges
- c  Only one subsequence of $p_n$ converges to $p$
- d  No subsequence of $p_n$ converges to $p$
- e  none of them

## Theorems for 300.

If $p_n \to p$, then

- a $p_n$ has a divergent Cauchy subsequence
- b $p_n$ is Cauchy and it might not converge to $p$
- c $p_n$ diverges
- d $p_n$ is Cauchy
- e $p_n$ converges to 0
- f $p_n$ converges for sure to 34.75
- g none of them

# Theorems for 400.

If $p_n$ converges to $p$, find the incorrect answer

- a $p_n$ might converge to $q$ where $q \neq p$.
- b $p_n$ might converge to all points
- c $p_n$ has a subsequence that converges
- d $p_n$ might diverge sometimes
- e $p_n$ is a Cauchy sequence always
- f none of them

# Counter Examples for 100.

A set open and closed in $\mathbb{R}$ usual topology

| | |
|---|---|
| a | $\{0, 1, 5\}$ |
| b | $\emptyset$ |
| c | $\mathbb{Q}$ |
| d | $\mathbb{I}$ |
| e | [0,1] |
| f | (0,1) |
| g | (0,1] |

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## Counter Examples for 200.

A sequence that converges to 500 different points

- a $p_n = 1/n,\quad \mathbb{R}$ usual top
- b $p_n = 1 - \frac{1}{n},\quad \mathbb{R}$ Sorgenfrey top
- c $p_n = (-1)^n,\quad \mathbb{R}$ discrete top
- d $p_n = n^2,\quad \mathbb{R}$ indiscrete top
- e $p_n = 0,\quad \mathbb{R}$ Sierpinksi top
- f none of them

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# Counter Examples for 300.

In this metric, balls are ONLY singletons

- a Taxi-Cab Driver metric
- b Usual metric
- c metric $d(x, y) = 0.$
- d Network metric
- e discrete metric
- f none of them

## Counter Examples for 400.

Which one is not true about rationals, (usual topology)?

- a $\mathbb{Q}$ is Countable
- b $\mathbb{Q}$ is dense in the irrationals
- c $\mathbb{Q}$ has the same cardinality as $\mathbb{N}$.
- d $\mathbb{Q}$ is not closed
- e $\mathbb{Q}$ is not open
- f $\mathbb{Q}$ is not bounded
- g The interior of $\mathbb{Q}$ is empty
- h The closure of $\mathbb{Q}$ contains the irrationals
- i $\mathbb{Q}$ is both open and closed

Compute for 100.

In $\mathbb{R}$ with the usual topology compute

$\overline{\mathbb{Q}} =$

Example : integers

Compute for 200.

In $\mathbb{R}$ with the usual topology compute

$$\left(\{0,1,2,3\}\right)^{O} = \quad$$

Example : universe

Compute for 300.

In $\mathbb{R}$ with the usual topology compute

$$\Big((0,1)\cup\{5,6,7\}\Big)^O =$$

Example : [1,10)

Compute for 400.

In $\mathbb{R}$ with the usual topology compute

$$\overline{(0,1) \cup (1,5) \cup (5,6)} = \square$$

Example : (-2,3)

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Top and Compc for 100.

In $\mathbb{R}^k$ with usual topology, what describes better Heine Borel Theorem?

- a Every set is bounded and closed
- b Compact implies bounded and closed
- c closed and bounded implies compact
- d compact iff closed and bounded
- e compact implies closed but not bounded
- f compact implies closed
- g none of them

## Top and Compc for 200.

A set $A$ is closed iff

- a $A' \notin \tau$
- b $A \in \tau'$
- c $A$ is not open
- d $A$ is open
- e none of them

# Top and Compc for 300.

$\{3, 15\}$ is

a two dots
b an interval
c an infinite set
d the numbers between 3 and 15 excluding both
e the numbers between 3 and 15 including both
f the numbers between 3 and 15 including only 3
g my lover

## Top and Compc for 400.

In $\mathbb{R}$ usual topology, the set $(0, 5]$

- a is empty
- b is closed but not open
- c is open and closed
- d is not closed but it is open
- e none of them