Channel Islands
CALIFORNIA STATE UNIVERSITY

California State University
INSTRUCTIONALLY RELATED ACTIVITIES
CHANNEL ISLANDS

Graduate Mathematics Seminar

The rational hull of a module

Gangyong Lee
Chungnam National University

$Q = End(E_S) = BiEnd({}_RE)$

Abstract: In commutative rings, it is well known that the *field of quotients* of an integral domain is the smallest field in which it can be embedded. It is also extended to a general commutative ring R. That is, the *total ring of quotients* of R consists of R and the inverse elements of non-zero-divisors of R. We denote the total ring of quotients of R by $S-1R$ where S is a set of all non-zero-divisors of R. Definitely, $S-1R$ is an overring including the based ring R. Several mathematicians studied the above construction to be extended to arbitrary rings. In the early 1930s, O. Ore introduced the Ore condition which is in connection with the question of extending beyond commutative rings the construction of a field of quotients, or more generally localization of a ring. Thus, if a ring satisfies the Ore condition then the ring can be embedded in its classical ring of quotients. It is developed by A. Goldie in the 1950's including the landmark Goldie theorem. However, there is an example that a domain cannot be embedding R into a division ring showing by A.I. Mal'cev. There is another way to find an overring of a given ring, called the *maximal right ring of quotients* of a ring. Any arbitrary ring can be embedded in its maximal right ring of quotients, which was more studied by K. Asano, R.E. Johnson, J. Lambek, G.D. Findlay, Y. Utumi and so on. In this talk, we introduce the notion of the rational hull of a module as in a general module theoretic setting of the maximal right ring of quotients. We provide characterizations of the rational hull of a module and investigate their properties. Especially, we prove that End$R(M)$ is embedded in End$R(E(M))$ for the rational hull e $E(M)$ of a right R-module M. Also, we show that the sufficient and necessary condition for the rational hull of the finite direct sum of modules to be the finite direct sum of the rational hulls of each module. (This is a joint work with Tung Nguyen and Xiaoxiang Zhang.).

***When*: Monday, February 3, 2020, 6:00 – 7:00 pm**
***Where*: CSUCI, Sierra Hall 2411**