Analytic And Computations

Theorems  Analytic  Computations  Computations2  Others
Theorems for 100.

State **Cauchy Integral Formula**

\[
\frac{n!}{2\pi i} \int_C \frac{f(z)}{(z - z_0)^{n+1}} \, dz = f^{(n)}(z_0)
\]

\[
\frac{2\pi i}{n!} \int_C \frac{f(z)}{(z - z_0)^n} \, dz = f^{(n)}(z_0)
\]

\[
\int_C \frac{f(z)}{(z - z_0)^n} \, dz = \frac{2\pi i}{n!} f^{(n)}(z_0)
\]

\[
\int_C \frac{f(z)}{(z - z_0)^{n+1}} \, dz = \frac{n!}{2\pi i} f^{(n)}(z_0)
\]

none of them
Theorems for 200.

What does Cauchy-Goursat say?

- If $f$ analytic, $C$ simple, then $\int_C f(z) \, dz = 0$
- If $f$ analytic, $C$ simple, then $\int_C f(z) \, dz > 0$
- If $f$ analytic, $C$ closed, then $\int_C f(z) \, dz < 0$
- If $f$ analytic, $C$ simple and closed, then $\int_C f(z) \, dz = 0$
- If $f$ continuous, $C$ Jordan, then $\int_C f(z) \, dz = 0$
- If $f$ bounded, $C$ Jordan, then $\int_C f(z) \, dz = 0$
Theorems for 300.

**Louville’s Theorem** states that

- Any polynomial has no real roots.
- Any polynomial has a real root.
- Any polynomial with complex coefficients has a real root.
- Any polynomial with complex coefficients has no complex roots.
- Any polynomial with complex coefficients has at least a complex root.
- Any polynomial with real coefficients has at least one complex root.
- none of above
De Moivre formula The roots of $z^n = re^{i\theta}$ are $z_k =$

$$r^{1/k} \left[ \cos \left( \frac{\theta + 2\pi kn}{k} \right) + i \sin \left( \frac{\theta + 2\pi k}{n} \right) \right], \quad k = 1, \ldots, n$$

$$r^{1/k} \left[ \cos \left( \frac{\theta - 2\pi k}{n} \right) + i \sin \left( \frac{\theta + 2\pi k}{n} \right) \right], \quad k = 1, \ldots, n$$

$$r \left[ \cos \left( \frac{\theta + 2\pi kn}{k} \right) + i \sin \left( \frac{\theta + 2\pi k}{n} \right) \right], \quad k = 1, \ldots, n$$

$$r^{1/k} \left[ \cos \left( \frac{\theta + 2\pi k}{n} \right) + i \sin \left( \frac{\theta + 2\pi k}{n} \right) \right], \quad k = 0, \ldots, n - 1$$
none of them
Analytic for 100.

What are the Cauchy Riemann Equations for $f = u + iv$?

$$v_{\theta} = ru_{r}, \quad u_{\theta} = -ru_{r}$$

$$v_{x} = u_{y}, \quad v_{y} = -u_{x}$$

$$v_{y} = u_{x}, \quad u_{x} = -v_{y}$$

$$v_{x} = v_{y}, \quad u_{x} = -v_{y}$$

$$v_{x} = v_{y}, \quad u_{x} = -u_{y}$$

none of them
Analytic for 200.

What does \textbf{holomorphic} mean?

Analytic
\[ u_{xx} + u_{yy} = 0 \]
Continuous
Differentiable
\[ u_x = v_y \]
There is a harmonic conjugate of \( v \)
none of them
Analytic for 300.

What does **harmonic** mean?

Analytic

\[ u_x x + u_y y = 0 \]

Continuous

Differentiable

\[ u_x = v_y \]

There is a harmonic conjugate of \( v \)

none of them
Analytic for 400.

The circle of radius 5 centered at $-i$ is

$$|z - i|^2 = 5$$
$$|z - i| = 25$$
$$|z - i|^2 = 25$$
$$(x - i)^2 + (y - i)^2 = 25$$
$$|z + i| = 5$$
$$e^{5i} = \theta$$

none of them
Computations for 100.

A parametrization of the circle of radius 5 centered at $-i$ oriented positively is

\[ z = 5e^{i\theta}, \quad \theta \in [0, \pi] \]
\[ z = e^{5i\theta}, \quad \theta \in [0, 2\pi] \]
\[ z = e^{-5i\theta}, \quad \theta \in [0, \pi] \]
\[ z = 5e^{i\theta}, \quad i \in [0, \pi] \]
\[ z = 5e^{i\theta}, \quad \pi \in [0, \theta] \]
\[ z = 5e^{i\theta}, \quad \theta \in [2\pi, 4\pi] \]

none of them
Computations for 200.

For $z = re^{i\Theta}$, $\Theta \in (-\pi, \pi)$ the Log of $z$ is defined to be

$$\ln r + i\Theta + 2\pi k, \quad k \in \mathbb{Z}$$
$$\ln r + i\Theta - 2\pi k, \quad k \in \mathbb{Z}$$
$$\ln r + i\Theta + \pi k, \quad k \in \mathbb{Z}$$
$$\ln r + i(\Theta + \pi k), \quad k \in \mathbb{Z}$$
$$\ln r + 2i(\Theta + \pi k), \quad k \in \mathbb{Z}$$
$$\ln r + i\Theta$$

none of them
Computations for 300.

For $z = re^{i\Theta}$, $\Theta \in (-\pi, \pi)$ the $\log(z)$ is defined to be

\[
\begin{align*}
\ln r + i(\Theta + 2\pi k), & \quad k \in \mathbb{Z} \\
\ln r + i\Theta - 2\pi k, & \quad k \in \mathbb{Z} \\
\ln r + i\Theta + \pi k, & \quad k \in \mathbb{Z} \\
\ln r + i(\Theta + \pi k), & \quad k \in \mathbb{Z} \\
\ln r + 2i(\Theta + \pi k), & \quad k \in \mathbb{Z} \\
\ln r + i\Theta & \text{none of them}
\end{align*}
\]
Computations for 400.

Which number represents $e^{-3i\pi/2}$?

$-i/2$

$2/i$

$i/1$

$-i/1$

$-1/2$

$0/2$

none of them
Computation2 for 100.

Value of $e^{i\pi/2}$

i
1
-1
0
-i
none of them
Computations2 for 200.

Value of $i^{2009}$

i
-i
-1
0
none of them
Computations2 for 300.

Value of principal branch of $\sqrt{i}$

\[
\sqrt{2}/2 - i\sqrt{2}/2 \\
i \\
i^2 \\
\sqrt{2}/2 + i\sqrt{2}/2 \\
none of them
\]
Computations2 for 400.

In hyperbolic functions?

\[ \sinh^2 z + \cosh^2 z = 1 \]
\[ \sinh^2 z - \cosh^2 z = 0 \]
\[ \sinh^2 z + \cosh^2 z = 2 \]
\[ - \sinh^2 z + \cosh^2 z = 1 \]

none of them
Others for 100.

What is a singularity?

A rarity
Something singular
A point where a function is zero
A point for which there is a neighborhood where the function is analytical except at the point
A point where the function vanishes
A point where the function is discontinuous
none of them
Others for 200.

What is a domain $D$?

Where the function is defined
Where the function is not zero
a set non-empty and connected
They usually tell me
Where I live
none of them
A function is entire if

- it is not broken
- it is analytic
- it is not rational
- it is differentiable everywhere
- it is analytic on the complex plane
- none of the above
Others for 400.

What is the Laplace equation?

\[ H_{xx}(x, y) + H_{yy}(x, y) = 0 \]

\[ \int_C f \, dz = 0 \]

\[ e^{i\theta} = \cos(\theta) + i \sin(\theta) \]

\[ \overline{f(z)} = f(\overline{z}) \]

\[ e^i + 1 = 0 \]